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Fuzzy Soft Multi Relation and Its Properties

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ABSTRACT
In this research works, the concept of fuzzy soft multi relation is motivated and its fundamental properties are to be studied. Here we present the inverse of a fuzzy soft multi relation and some fundamental properties in regards to with these ideas are investigated. Also, we examine reflexive, symmetric and transitive fuzzy soft multi relations with numerical samples and study the effect of inverse on these various types of fuzzy soft multi relations.

KEYWORDS
Soft set — Fuzzy soft multi set — Fuzzy soft multi relation — Inverse.

1. Introduction

The majority of the problems in software engineering, computer science, medi-cal science, economics, environments and so on have different uncertainties. In 1999, Molodstov [1] started the idea of soft set theory as a numerical model for managing uncertainties. Research works on soft set theory are advancing quick-ly. Maji et al. [2]-[3] characterized a few operations on soft set theory and stud-ied its application in decision making problem. In light of the examination of a few operations on soft sets, Ali et al. [4] introduced some new mathematical operations for soft sets. Consolidating soft sets with fuzzy sets [5], Maji et al. [6] characterized fuzzy soft sets. Alkhazaleh and other [7] - [11] as a speculation of Molodtsov's soft set, displayed the meaning of a soft multiset and concentrated on its fundamental operations, for example, union, intersection, complement, so on furthermore, characterized the some algebraic and topological structure on soft multi sets. In 2012 Alkhazaleh and Salleh [12] presented the idea of fuzzy soft multi set theory and as of late Mukherjee and Das [13]-[14] considered some new results on these sets and characterized the topological structure on fuzzy soft multisets. Also, they studied the application of fuzzy soft multisets in their papers [15]-[16].

Actually, every one of these ideas having a decent application in different con-trols and genuine issues is presently getting force. Yet, it is seen that every one of these speculations has their own challenges that is the reason in this paper, we are going to study the idea of relations in the fuzzy soft multi set, which is another new numerical device for managing instabilities and obscure ideas.

The idea of fuzzy relation on a set was characterized by Zadeh [17] and a few creators have thought of it as further. Recently Mukherjee and Das [18] introduced the concept of relation on intuitionistic fuzzy soft multisets. In this paper, we characterize the product of two fuzzy soft multi sets and present the idea of fuzzy soft multi relation. Also, we present the inverse of a fuzzy soft multi relation and some fundamental properties in regards to with these ideas are investigated. Likewise, we examine their properties and talk about reflexive, symmetric and transitive fuzzy soft multi relations with illustrations and study the effects of in-verse on these various types of fuzzy soft multi relations.

2. Preliminary Notes

In this area, we review some fundamental thoughts on soft set theory and fuzzy soft multi set theory. Let U be a beginning universe and E be a set of parameters. Let $P(U)$ indicates the power set of U and $A \subseteq E$.

Definition 2.1[9]
A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.2[[3], [14]]
Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} FS(U_i)$ where $FS(U_i)$ denotes the set of all fuzzy subsets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F,A) is called a fuzzy soft multi set over U, where $F : A \to U$ is a mapping given by $\forall e \in A$,

$$F(e) = \left(\left\{ \frac{u}{\mu_{F(e)}(u)} : u \in U_i \right\} : i \in I \right)$$

Definition 2.3[3]
For any FSM-set (F,A), where $A \subseteq E$ and E is a situated of parameters. A couple $(e_{U_{i,j}}, F_{e_{U_{i,j}}})$ is known as a U_i-fuzzy soft multiset part (U_i-FSMS-part) of (F,A) on U, $\forall e_{U_{i,j}} \in a$ and $F_{e_{U_{i,j}}} \subseteq F(A)$ is an approximate value set, where $a \in A, i, j \in A$.

Definition 2.4[3] A fuzzy soft multi set (F,A) over U is called a null fuzzy soft multi set, denoted by $(F,A)_\varphi$, if all the fuzzy soft multi set parts of (F,A) equals φ.

Definition 2.5[3]
A fuzzy soft multi set (F,A) over U is called an absolute fuzzy soft multi set, de-noted by $(F,A)_U$, if $(e_{U_{i,j}}, F_{e_{U_{i,j}}}) = U_i, \forall i$.

Definition 2.6[14]
A fuzzy soft multi set (F, A) over U is called fuzzy soft multi subset of a fuzzy soft multi set (G,B) if

(a) $A \subseteq B$ and

(b) $\forall e \in A, F(e) \subseteq G(e) \Leftrightarrow \mu_{F(e)}(u) \leq \mu_{G(e)}(u), \quad \forall u \in U_i,\ i \in I$.

This relationship is denoted $(F,A) \in \tilde{\subseteq}(G,B)$

Definition 2.7 [10]
Two fuzzy soft multi sets (F,A) and (G,B) over U are called equal if (F,A) is FSM-subset of (G,B) and (G,B) is fuzzy soft multi subset of (F,A).

Definition 2.8[14]
The complement of a fuzzy soft multi set (F,A) over U is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where $\forall e \in A$

$$F^c(e) = \left(\left\{ \frac{u}{1-\mu_{F(e)}(u)} : u \in U_i \right\} : i \in I \right)$$

.

3. Fuzzy Soft Multi Relations

In this segment, we characterize product of two fuzzy soft multi sets and present the idea of relations in fuzzy soft multi sets. The thoughts of null relation and ab-solute relation are to be characterized.

3.1 Product of Two fuzzy soft multi sets

The product $(F,A) \times (G,B)$ of two fuzzy soft multi sets (F,A) and (G,B) over U is a fuzzy soft multi set $(H, A \times B)$ where H is mapping given by $H : A \times B \to U$ and $\forall (a,b) \in A \times B$

$$\begin{aligned} H(a,b) &= \cap(F(a),G(b)) \\ &= \left(\left\{ \frac{u}{\min\{\mu_{F(a)}(u), \mu_{G(b)}(u)\}} : u \in U_i \right\} : i \in I \right) \end{aligned}$$

Table 1. Fuzzy soft multi set (F,A)

(F,A)	a_1	a_2	a_3
h_1	0.7	0.5	0.3
h_2	0.3	0.1	0.7
h_3	0.7	0.3	0.5
c_1	0.7	0.3	0.6
c_2	0.4	0.4	0.4

Table 2. Fuzzy soft multi set (G,B)

(G,B)	b_1	b_2
h_1	0.5	0.3
h_2	0.7	0.5
h_3	0.3	0.5
c_1	0.4	0.7
c_2	0.7	0.4

Example 3.2
Give us a chance to consider two universes $U_1 = \{h_1, h_2, h_3\}$, $U_2 = \{c_1, c_2\}$. Let $\{E_{U_1}, E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}\}$. Let $U = \prod_{i=1}^{2} FS(U_i)$, $E = \prod_{i=1}^{2} E_{U_i}$ and $A, B \subseteq E$, such that $A = \{a_1 = (e_{U_1,1}, e_{U_2,1}), a_2 = (e_{U_1,1}, e_{U_2,2}), a_3 = (e_{U_1,2}, e_{U_2,2})\}$ and $B = \{b_1 = (e_{U_1,2}, e_{U_2,1}), b_2 = (e_{U_1,2}, e_{U_2,2})\}$

3.2 Fuzzy Soft Multi Relation

Let (F,A) be a fuzzy soft multi set over U. Then a fuzzy soft multi relation R on (F,A) will be a fuzzy soft multi subset of the product set $(F,A) \times (F,A)$ and is characterized as a couple $(R, A \times A)$, where R is mapping given by $R : A \times A \to U$. The collection of all fuzzy soft multi relations R on (F,A) over U is indicated by $FSMR_U(F,A)$.

Example 3.4
Consider the fuzzy soft multi set (F,A) given in Table 1, then a fuzzy soft multi relation R on (F,A) is given by in Table 4.

Table 3. The product set $(H, A \times B)$:

	(a_1,b_1)	(a_1,b_2)	(a_2,b_1)	(a_2,b_2)	(a_3,b_1)	(a_3,b_2)
h_1	0.5	0.3	0.5	0.3	0.3	0.3
h_2	0.3	0.3	0.1	0.1	0.7	0.5
h_3	0.3	0.5	0.3	0.3	0.3	0.5
c_1	0.4	0.7	0.3	0.3	0.6	0.6
c_2	0.4	0.4	0.4	0.4	0.4	0.4

Table 4. Fuzzy soft multi relation R

R	(a_1,a_1)	(a_1,a_2)	(a_1,a_3)	(a_2,a_1)	(a_2,a_2)	(a_2,a_3)	(a_3,a_1)	(a_3,a_2)	(a_3,a_3)
h_1	0.6	0.4	0.3	0.5	0.4	0.3	0.3	0.3	0.3
h_2	0.2	0.1	0.3	0.1	0.1	0.1	0.3	0.1	0.6
h_3	0.5	0.3	0.5	0.3	0.3	0.3	0.5	0.3	0.5
c_1	0.7	0.3	0.6	0.3	0.3	0.3	0.2	0.3	0.6
c_2	0.4	0.4	0.4	0.2	0.4	0.4	0.2	0.4	0.2

Definition 3.5
Let R_1, $R_2 \in FSMR_U(F,A)$, then we define for $(a,b) \in A\times A$,
(i) $R_1 \leq R_2$ if and only if $R_1(a,b) \subseteq R_2(a,b)$, for $(a,b) \in A\times A$.
(ii) $R_1 \vee R_2$ as $(R_1 \vee R_2)(a,b) = R_1(a,b) \cup R_1(a,b)$, where $\cup$ denotes the fuzzy union.
(iii) $R_1 \wedge R_2$ as $(R_1 \wedge R_2)(a,b) = R_1(a,b) \cap R_2(a,b)$, where $\cap$ denotes the fuzzy intersection.
(iv) R_1^c as $R_1^c(a,b) = C[R_1(a,b)]$, where C denotes the fuzzy complement.

Result 3.6
For three fuzzy soft multi relations R_1, R_2, $R_3 \in FSMR_U(F,A)$, the following properties hold:

1. De Morgan Laws:
   (a) $(R_1 \vee R_2)^C = R_1^C \wedge R_2^C$.
   (b) $(R_1 \wedge R_2)C = R_1^C \vee R_2^C$.
2. Associative Laws:
   (a) $R_1 \vee (R_2 \vee R_3) = (R_1 \vee R_2) \vee R_3$
   (b) $R_1 \wedge (R_2 \wedge R_3) = (R_1 \wedge R_2) \wedge R_3$
3. Distributive Laws:
   (a) (a) $R_1 \wedge (R_2 \vee R_3) = (R_1 \wedge R_2) \vee (R_1 \wedge R_3)$
   (b) (b) $R_1 \vee (R_2 \wedge R_3) = (R_1 \vee R_2) \wedge (R_1 \vee R_3)$

Definition 3.7
A null fuzzy soft multi relation $R_\Phi \in IFSMR(F,A)$ is defined as $R_\Phi = (R_\Phi, A\times A)_\Phi$ where $(R_\Phi, A\times A)_\Phi$ is the null fuzzy soft multi set and an absolute fuzzy soft multi relation $R_U \in FSMR_U(F,A)$ is defined as $R_U = (R_U, A\times A)_U$ where $(R_U, A\times A)_U$ is the absolute fuzzy soft multi set.

Remark 3.8
For any fuzzy soft multi relation $R \in FSMR_U(F,A)$, we have

(i) $R \vee R_\Phi = R$

(ii) $R \wedge R_\Phi = R_\Phi$

(iii) $R \vee R_U = R_U$

(iv) $R \wedge R_U = R$

4. Inverse of fuzzy soft multi relation

In this section, we introduced the inverse of fuzzy soft multi relation and study some important properties regarding the above concept and some basic properties regarding with these concepts are investigated.

Definition 4.1
Let $R \in IFSMR(F,A)$ be a fuzzy soft multi relation on (F,A). Then R^{-1} is defined as $\forall, b \in A$, $R^{-1}(a,b) = R(b,a)$, i.e.

$$\mu_{R^{-1}(a,b)}(u) = \mu_{R(a,b)}(u),\ \forall i \in I$$

Example 4.2
If we consider the fuzzy soft multi relation R as in Table 4, then R^{-1} is given by as in Table 5.

Proposition 4.3
If R be a fuzzy soft multi relation on (F,A), then R^{-1} is also a fuzzy soft multi relation on (F,A).

Proof:
$R^{-1}(a,b) = R(b,a) \subseteq F(b) \cap F(a) = F(a) \cap F(b)\ \forall a,b \in A$, i.e. $R^{-1}(a,b) \subseteq F(a) \cap F(b)\ \forall a,b \in A$, implies R^{-1} is a FSM-subset of the product set $(F,A)\times(F,A)$ and hence R^{-1} is a fuzzy soft multi relation on (F,A).

Proposition 4.4
If R_1 and R_2 be two fuzzy soft multi relations on (F,A), then

1. $(R_1^{-1})^{-1} = R_1$
2. $R_1 \subseteq R_2 \rightarrow R_1^{-1} \subseteq R_2^{-1}$

Proof:
Let us consider R_1 and R_2 be two fuzzy soft multi relations on (F,A), then $\forall a,b \in A$,

1. $(R_1^{-1})^{-1}(a,b) = R_1^{-1}(b,a) = R_1(a,b)$. Hence $(R_1^{-1})^{-1} = R_1$
2. $R_1(a,b) \subseteq R_2(a,b) \Rightarrow R_1^{-1}(b,a) \subseteq R_2^{-1}(b,a) \Rightarrow R_1^{-1} \subseteq R_2^{-1}$

Proposition 4.5
If R_1 and R_2 be two fuzzy soft multi relations on (F,A), then
(a) $(R_1 \vee R_2)^{-1} = R_1^{-1} \vee R_2^{-1}$

Table 5. R^{-1}

R^{-1}	(a_1,a_1)	(a_1,a_2)	(a_1,a_3)	(a_2,a_1)	(a_2,a_2)	(a_2,a_3)	(a_3,a_1)	(a_3,a_2)	(a_3,a_3)
h_1	0.6	0.5	0.3	0.4	0.4	0.3	0.3	0.3	0.3
h_2	0.2	0.1	0.3	0.1	0.1	0.1	0.3	0.1	0.6
h_3	0.5	0.3	0.5	0.3	0.3	0.3	0.5	0.3	0.5
c_1	0.7	0.3	0.2	0.3	0.3	0.3	0.6	0.3	0.6
c_2	0.4	0.2	0.2	0.4	0.4	0.4	0.4	0.4	0.2

Table 6. Fuzzy soft multi-set (F,A)

(F,A)	**a**	**b**	**c**
h_1	1	1	1
h_2	1	1	1
h_3	1	1	1
c_1	1	1	1
c_2	1	1	1

(b) $(R_1 \wedge R_2)^{-1} = R_1^{-1} \wedge R_2^{-1}$

Proof:
Let us consider R_1 and R_2 be two fuzzy soft multi relations on (F,A), then $\forall a,b \in A$

$$(R_1 \vee R_2)^{-1}(a,b) = (R_1 \vee R_2)(b,a) = R_1(b,a) \vee R_2(b,a)$$

$$= R_1^{-1}(a,b) \vee R2^{-1}(a,b) = (R_1^{-1} \vee R_2^{-1})(a,b)$$

Hence $(R_1 \vee R_2)^{-1} = R_1^{-1} \vee R_2^{-1}$. Similarly we can be proved the other.

5. Various Types of fuzzy soft multi relations and Their Properties

Definition 5.1
A fuzzy soft multi relation $R \in FSMR_U(F,A)$ is known as reflexive if $\mu_{R(a,a)}(u) = 1, \forall u \in U_i, \forall i \in I$ and $\forall a \in A$.

Example 5.2
Give us a chance to consider two universes $U_1 = \{h_1,h_2,h_3\}$, $U_2 = \{c_1,c_2\}$. Let $\{E_{U_1},E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1} = \{e_{U_1,1},e_{U_1,2},e_{U_1,3}\}$ and $E_{U_2} = \{e_{U_2,1},e_{U_2,2}\}$. Let $U = \prod_{i=1}^{2} FS(U_i)$, $E = \prod_{i=1}^{2} E_{U_i}$ and $A \subseteq E$, such that $A = \{a = (e_{U_1,1},e_{U_2,1}), b = (e_{U_1,1},e_{U_2,2}), c = (e_{U_1,2},e_{U_2,1})\}$
Then the fuzzy soft multi relation $(R, A\times A)$ on (F,A) as in Table 6, is a reflexive fuzzy soft multi relation on (F,A).

Proposition 5.3
R is reflexive on (F,A) if and only if R^{-1} is reflexive.

Proof:
Let us consider R is reflexive relation on (F,A). Then $\forall (a,a) \in A\times A$

$$\mu_{R^{-1}(a,a)}(u) = \mu_{R(a,a)}(u) = 1,\ \forall u \in U_i,\ \forall i \in I.$$

Thus R^{-1} is reflexive relation on (F,A).
Conversely, let R^{-1} is reflexive relation on (F,A). Then $\forall (a,a) \in A\times A$

$$\mu_{R(a,a)}(u) = \mu_{R^{-1}(a,a)}(u) = 1,\ \forall u \in U_i,\ \forall i \in I.$$

Thus R is reflexive relation on (F,A).

Definition 5.4
A fuzzy soft multi relation $R \in FSMP_U(F,A)$ is said to be symmetric if

$$\mu_{R(a,b)}(u) = \mu_{R(b,a)}(u),\ \forall u \in U_i,\ \forall i \in I \text{ and } \forall a \in A.$$

Example 5.5
Consider the fuzzy soft multi set (F,A) as in Table 1. Then the relation R be characterized as in Table 8, is a symmetric fuzzy soft multi relation on (F,A).

Proposition 5.6
R is symmetric fuzzy soft multi relation on (F,A) if and only if R^{-1} is symmetric.

Proof:
Let us consider R is symmetric fuzzy soft multi relation on (F,A). Then $\forall (a,b) \in A\times A$

$$\mu_{R^{-1}(a,b)}(u) = \mu_{R(b,a)}(u) = \mu_{R(a,b)}(u) = \mu_{R^{-1}(b,a)}(u), \quad \forall u \in U_i, \forall i \in I$$

i.e. $R^{-1}(a,b) = R(b,a) = R(a,b) = R^{-1}(b,a)$. Thus R^{-1} is symmetric fuzzy soft multi relation on (F,A).
Conversely, let R^{-1} is symmetric fuzzy soft multi relation on (F,A). Then $\forall (a,b) \in A\times A$

$$\mu_{R^{-1}(a,b)}(u) = \mu_{R^{-1}(b,a)}(u) = \mu_{R^{-1}(a,b)}(u) = \mu_{R(b,a)}(u), \quad \forall u \in U_i, \forall i \in I$$

i.e. $R(a,b) = R^{-1}(b,a) = R^{-1}(a,b) = R(b,a)$.
Thus R is symmetric fuzzy soft multi relation on (F,A).

Table 7. Reflexive fuzzy soft multi relation R

R	**(a,a)**	**(a,b)**	**(a,c)**	**(b,a)**	**(b,b)**	**(b,c)**	**(c,a)**	**(c,b)**	**(c,c)**
h_1	1	0.3	0.1	0.1	1	0.4	0.4	1	1
h_2	1	0.5	0.7	0.6	1	0.1	0.1	0	1
h_3	1	0.8	0.9	1	1	0.3	0.3	0.9	1
c_1	1	0.7	0.8	1	1	0.3	0.3	0.8	1
c_2	1	0.1	0.5	1	1	0.4	0.4	0	1

Table 8. Symmetric fuzzy soft multi relation R

R	(a_1,a_1)	(a_1,a_2)	(a_1,a_3)	(a_2,a_1)	(a_2,a_2)	(a_2,a_3)	(a_3,a_1)	(a_3,a_2)	(a_3,a_3)
h_1	0.6	0.4	0.3	0.4	0.4	0.3	0.3	0.3	0.3
h_2	0.2	0.1	0.3	0.1	0.1	0.1	0.3	0.1	0.6
h_3	0.5	0.3	0.5	0.3	0.3	0.3	0.5	0.3	0.5
c_1	0.7	0.3	0.6	0.3	0.3	0.3	0.6	0.3	0.6
c_2	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.2

Proposition 5.7
R is symmetric fuzzy soft multi relation on (F,A) if and only if $R^{-1} = R$.

Proof:
Let us consider R is symmetric fuzzy soft multi relation on (F,A). Then $\forall (a,b) \in A \times A$

$$\mu_{R^{-1}(a,b)}(u) = \mu_{R(b,a)}(u) = \mu_{R(a,b)}(u), \quad \forall u \in U_i,\ \forall i \in I$$

i.e. $R^{-1}(a,b) = R(b,a) = R(a,b)$. Thus $R^{-1} = R$.
Conversely, let $R^{-1} = R$. Then $\forall (a,b) \in A \times A$

$$\mu_{R(a,b)}(u) = \mu_{R^{-1}(a,b)}(u) = \mu_{R(b,a)}(u), \quad \forall u \in U_i,\ \forall i \in I$$

i.e. $R(a,b) = R^{-1}(a,b) = R(b,a)$.
Thus R is symmetric fuzzy soft multi relation on (F,A).

Definition 5.8
Let $R_1, R_2 \in FSMR_U(F,A)$ be two fuzzy soft multi relations on (F,A). Then the composition of R_1 and R_2, denoted by $R_1 \circ R_2$ and is defined by $R_1 \circ R_2 = (R_1 \circ R_2, A \times A)$ where $R_1 \circ R_2 : A \times A \to U$ is defined as $\forall u \in U_i$, $\forall i \in I$ and $\forall a,b,c \in A$

$$\mu_{R_1 \circ R_2(a,b)}(u) = \max_c\{\min(\mu_{R_1(a,c)(u)}, \mu_{R_2(c,b)(u)})\}$$

Proposition 5.9
If R_1 and R_2 be two fuzzy soft multi relations on (F,A), then $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$

Proof:
Let us consider R_1 and R_2 be two fuzzy soft multi relations on (F,A), then $\forall a,b \in A$

$$\begin{aligned}
\mu_{R_2^{-1} \circ R_1^{-1}(a,b)}(u) &= \max_c\{\min(\mu_{R_2^{-1}(a,c)}(u), \mu_{R_1^{-1}(c,b)}(u))\} \\
&= \max_c\{\min(\mu_{R_2(c,a)}(u), \mu_{R_1(b,c)}(u))\} \\
&= \max_c\{\min(\mu_{R_1(b,c)}(u), \mu_{R_2(c,a)}(u))\} \\
&= \mu_{R_1 \circ R_2(b,a)}(u) \\
&= \mu_{(R_1 \circ R_2)^{-1}(a,b)}(u), \quad \forall u \in U_i,\ \forall i \in I
\end{aligned}$$

Hence $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.

Proposition 5.10
If R is symmetric fuzzy soft multi relation on (F,A) then $R \circ R$ is symmetric on (F,A).

Proof:
Let us consider R is symmetric fuzzy soft multi relation on (F,A). Then $R^{-1} = R$. Now $(R \circ R)^{-1} = R^{-1} \circ R^{-1} = R \circ R$. Hence $R \circ R$ is symmetric on (F,A).

Proposition 5.11
If R_1 and R_2 be two symmetric fuzzy soft multi relations on (F,A), then $R_1 \circ R_2$ is symmetric on (F,A) if and only if $R_1 \circ R_2 = R_2 \circ R_1$.

Proof:
R_1 and R_2 be two symmetric fuzzy soft multi relations on (F,A) implies that, $R_1^{-1} = R_1$ and $R_2^{-1} = R_2$. Now $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.
$R_1 \circ R_2$ is symmetric on (F,A) implies $R_1 \circ R_2 = (R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1} = R_2 \circ R_1$.
Conversely, $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1} = R_2 \circ R_1 = R_1 \circ R_2$.
So $R_1 \circ R_2$ is symmetric on (F,A).

Table 9. Transitive fuzzy soft multi relation R on (F,A)

R	(a,a)	(a,b)	(a,c)	(b,a)	(b,b)	(b,c)	(c,a)	(c,b)	(c,c)
h_1	0.5	0.4	0.4	0.3	0.4	0.3	0.3	0.4	0.5
h_2	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.3
h_3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
c_1	0.4	0.1	0.1	0.1	0.3	0.1	0.1	0.1	0.4
c_2	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

Definition 5.12
A fuzzy soft multi relation $R \in FSMR_U(F,A)$ is said to be a transitive if and only if $R \circ R \subseteq R$.

Example 5.13
Consider the fuzzy soft multi set (F,A) as in Table 1. Then the relation R be characterized as in Table 9, is a transitive fuzzy soft multi relation on (F,A).

Proposition 5.14
R is transitive relation on (F,A) if and only if R^{-1} is transitive relations on (F,A).

Proof:
Let us consider R is transitive relation on (F,A), then $\forall a,b \in A$

$$\begin{aligned}
\mu_{R^{-1}(a,b)}(u) &= \mu_{R(b,a)}(u) \geq \mu_{R\circ R(b,a)}(u) \\
&= \max_c\{\min(\mu_{R(b,c)}(u), \mu_{R(c,a)}(u))\} \\
&= \max_c\{\min(\mu_{R(c,a)}(u), \mu_{R(b,c)}(u))\} \\
&= \max_c\{\min(\mu_{R^{-1}(a,c)}(u), \mu_{R^{-1}(c,b)}(u))\} \\
&= \mu_{R^{-1}\circ R^{-1}(a,b)}(u), \quad \forall u \in U_i,\ \forall i \in I
\end{aligned}$$

Hence R^{-1} is transitive relations on (F,A).
Conversely, let R^{-1} is transitive relation on (F,A). Then $\forall (a,a) \in A \times A$,

$$\begin{aligned}
\mu_{R(a,b)}(u) &= \mu_{R^{-1}(b,a)}(u) \geq \mu_{R^{-1}\circ R^{-1}(b,a)}(u) \\
&= \max_c\{\min(\mu_{R^{-1}(b,c)(u)}, \mu_{R^{-1}(c,a)(u)})\} \\
&= \max_c\{\min(\mu_{R^{-1}(c,a)(u)}, \mu_{R^{-1}(b,c)(u)})\} \\
&= \max_c\{\min(\mu_{R(a,c)(u)}, \mu_{R(c,b)(u)})\} \\
&= \mu_{R\circ R(a,b)}(u), \quad \forall u \in U_i,\ \forall i \in I
\end{aligned}$$

Hence R is transitive relations on (F,A).

6. Conclusion

In this examination work, the concept of fuzzy soft multi relation is explored and its fundamental properties are to be studied. Also, we present the inverse of fuzzy soft multi relation and some fundamental properties in regards to with these ideas are investigated. Also, we examine reflexive, symmetric and transitive fuzzy soft multi relations with numerical samples and study the effect of inverse on these various types of fuzzy soft multi relations.

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